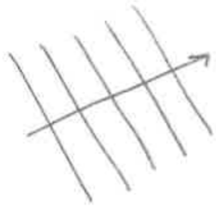
Optics Review

I. Geometric Optics



Wavefronts are defined $\perp$ to Ray

Wavefront

Ray

Geometric optics Approximations: (1) Rays Move in straight lines in homogenous Media (& can curve in inhomogenous media) (2) Rays are parameterized by position x & angle of propagation Θ

⚠ Frequency does not change → λ & propagation speed change

$$\boxed{c = \lambda\nu \Rightarrow \frac{c}{n} = \frac{\lambda}{n}\nu}$$ → $n \equiv$ refractive index → "Optical density"

↳ light travels slower in higher refractive index material

Snell's Law: $\boxed{\dfrac{n_1}{n_2} = \dfrac{\sin\theta_2}{\sin\theta_1}}$ ⇒ Rays bend at interfaces of varying n

↳ higher idex contrast ⇒ light rays bend More

↳ Snell's law is the result of the principle of least time → Fermat's principle → Snell's Law minimizes the optical path length!

Fermat's Principle: The path that the ray follows is such that the value of the path integral of the refractive index, i.e. $OPL = \int_{path} n(r)ds$, is smaller than all other possible paths.

Critical Angle: $\boxed{n_1 \sin\Theta_{crit} = n_2}$ → Above this angle, Total Internal Reflection occurs → light cannot propogate through interface but energy must be conserved ⇒ light must Reflect

↳ Important for fiber optics

Graded Index (GRIN) Waveguide: $n(x) = n_{max} - \gamma x^2$

ABCD Matrix: describes how system transforms Ray

$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_{in} \\ x_{out} \end{pmatrix} \quad \begin{cases} A = \dfrac{\partial x_{out}}{\partial x_{in}} \equiv \text{Spatial Magnification} \\ B = \partial x_{out} / \partial \theta_{in} \; ; \; C = \dfrac{\partial \theta_{out}}{\partial x_{in}} \\ D = \dfrac{\partial \theta_{out}}{\partial \theta_{in}} \equiv \text{angular magnification} \end{cases}$$

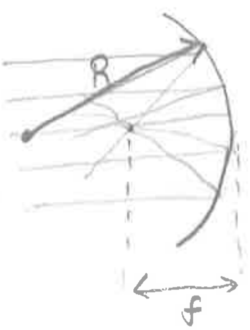
e.g. Snell's Law (Paraxial Approximation)

$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix}$$

Law of Reflection: $\theta_{out} = -\theta_{in} \rightarrow$ A ray departing P in the direction θ is reflected symmetrically at the same angle θ.

Curved Mirrors: (i.e. spherical) $f \approx \frac{-R}{2}$ Mirror oriented towards left so $R<0$.

⚠ Derive by setting taylor expanded circle equation equal to parabola $s = \frac{x^2}{4f}$



ABCD Matrix: $\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix}$

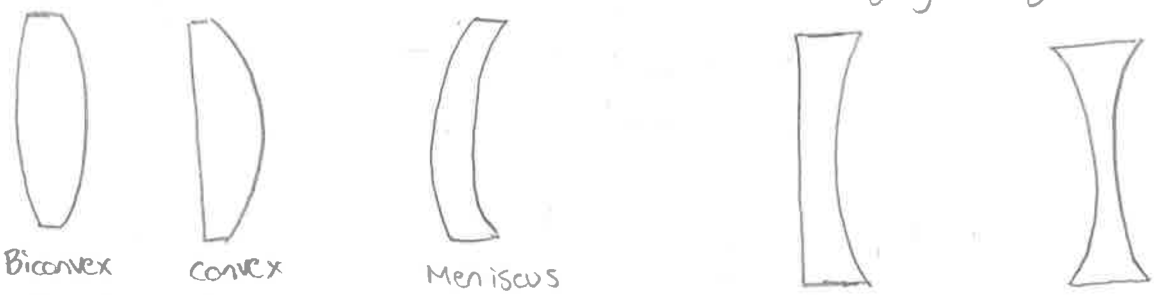
Note: $f>0$ for a positive lens

Perfect focusing inside refractive Material $\longrightarrow$ Ellipsoid!

$$\left(s - \frac{n}{n+1} f\right)^2 + \frac{n^2}{n^2-1} x^2 = \left(\frac{n}{n+1} f\right)^2$$

Types of Lenses: Converging $\Rightarrow$ Positive focal length
Diverging $\Rightarrow$ Negative focal length

Converging (Positive): | Diverging (Negative)



Biconvex: $R_1 > 0$, $R_2 < 0$

Convex: $R_1 = \infty$, $R_2 < 0$

Meniscus: $R_1 > 0$, $R_2 > 0$

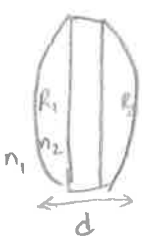
Plano-concave: $R_1 = \infty$, $R_2 > 0$

Biconcave: $R_1 < 0$, $R_2 > 0$

Transfer Matrix for Refraction from spherical surface:

$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{n_2 - n_1}{R n_2} & \frac{n_1}{n_2} \end{pmatrix} \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix} = A(R) \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix}$$

Thick lens:



$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = A(R_2) \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} A(R_1) \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix}$$

Thin lens: $(d \to 0)$

$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = A(R_2) A(R_1) \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \underbrace{\frac{n_1 - n_2}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{-1/f} & 1 \end{pmatrix} \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix}$$

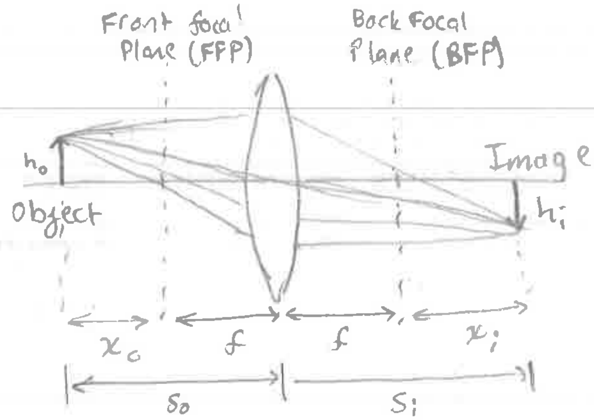
Lensmaker's Formula: (for thin lenses)

$$\boxed{\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Fourier Transform w/ Lens:

$$\begin{pmatrix} x_{out} \\ \theta_{out} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}}_{\text{Propagate focal len}} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\text{Thin lens}} \underbrace{\begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}}_{\text{Propagate focal le}} \begin{pmatrix} x_{in} \\ \theta_{in} \end{pmatrix} \Rightarrow \boxed{\begin{aligned} x_{out} &= f\theta_{in} \\ \theta_{out} &= -\frac{1}{f} x_{in} \end{aligned}}$$

Image Formation:



⚠ The object & image planes are "conjugate" planes

Lateral Magnification: $\boxed{M_T \equiv \frac{h_i}{h_o}} = -\frac{f}{x_o} = -\frac{x_i}{f}$

Imaging Condition: $\boxed{x_o x_i = f^2} \Rightarrow \boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$

Magnification of 1 occurs when $s_o = 2f = s_i$

⚠ Cannot focus object closer to lens than focal length f

⚠ When object is closer to the lens than 1 focal length, we see magnified virtual image behind it

Back Focal Plane: Z position where ray entering parallel to optical axis passes through optical axis.

Front Focal Plane: Z position where a ray that exits parallel to optical axis initially crosses optical axis

⚠ Thick systems have multiple focal planes ⟶ Principle planes

Virtual Image → Image appears closer to lens than focal length
Real Image → Image appears beyond focal length → Projectable

Principle Planes are located where a line formed by an incoming ray parallel to the optical axis intersects with a line formed by a ray exiting the lens (As well as the reverse.

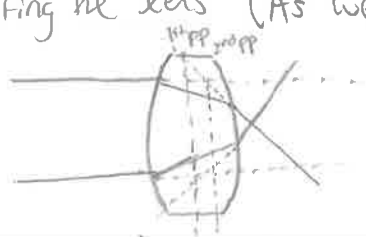
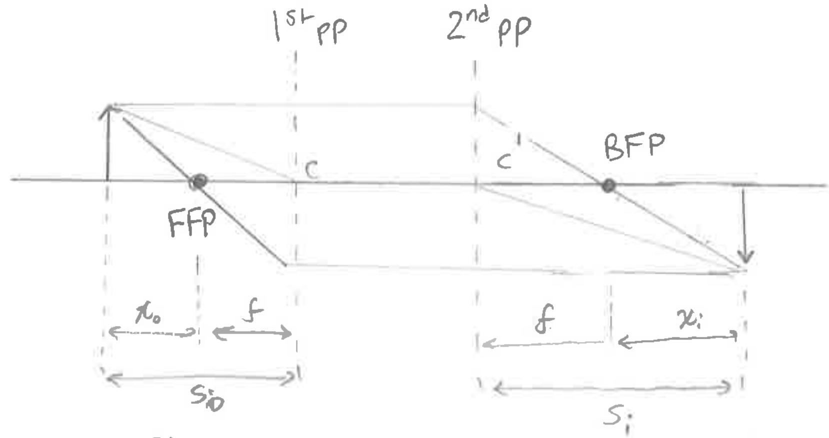


Image Formation w/ Composite Elements:



↳ Similar Triangle Arguments Still Apply i.e.,

$$x_o x_i = f^2 \; ; \; \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \; ; \; M_T = \frac{-x_i}{f} = -\frac{f}{x_o} = -\frac{s_i}{s_o} \; ; \; M_A = \frac{1}{M_T}$$

Depth of Focus: Range of image distances for which object is in focus
↳ determined by: (1) Distance to subject (2) Focal Length (3) Aperture (f stop)

Depth of Field: Range of object distances for which object is in focus

Numerical Aperture: $\boxed{NA = n \sin\theta}$

f-number: $\boxed{f/\# = \frac{1}{2NA}}$ (Note: $\frac{f}{\#}$ = Diameter of opening)

Circle of Confusion: largest diameter circular spot we are willing to tolerate in our system

Chief Ray: starts at object and passes through center of aperture stop

Marginal Ray: Most extreme angle allowed by optical system

→ CR + MR define angular acceptance of spherical ray bundle originating from off axis object.

Field Stop: Constrains angular range of Chief Rays
↳ defines the Field of View

⚠ Numerical Aperture defined as angle formed by Marginal rays that just make it through the Aperture stop.

Imaging Systems

Easy treatment of composite lens systems:
- ↳ Treat each lens separately, find the virtual image using $\frac{1}{S_o}+\frac{1}{S_i}=f$
- ↳ Virtual image becomes object for next lens
- ↳ Multiply magnifications at each step to get total magnification
  $m_{tot} = m_1 m_2 \cdots m_N$

sensor sensitivity? spectral distribution

Pixel Intensity has wavelength dependence: $I = k\int_{-\infty}^{\infty} q(\lambda)P(\lambda)d\lambda$

Signal intensity: $\langle I \rangle = \alpha \bar{n} q / \Delta t$

Photographic Demosaicing: Need twice as many Green pixels to render for the human eye $\Rightarrow$ Need 2x the resolution!

R	G	R	G	R	G	R	G
G	B	G	B	G	B	G	B

⚠ Why are two lenses better than 1? → Better Numerical Aperture and ability to exchange objective lens

Parts of a microscope: eyepiece, objective, condenser, light source, light conditioner

Contrast: measure of intensity fluctuations across image $:= \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$

Dark field Microscopy: ① Start w/ image w/ bright background ② Fourier transform image ③ Remove DC/Low freq components (ie high pass filter) ④ Inverse Fourier transform to retrieve higher contrast image

Bright Field Microscopy → Produces low contrast images of phase objects
- ↳ Use phase imaging to improve contrast!

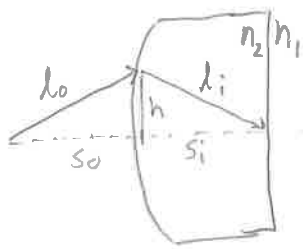
Aberrations

Recall: $\sin\theta = \underset{\substack{\uparrow \\ 1^{st}\ \text{order}}}{\theta} - \underset{\substack{\uparrow \\ 3^{rd}\ \text{order}}}{\frac{\theta^3}{3!}} + \underbrace{\frac{\theta^5}{5!} - \cdots}_{\text{Higher order}}$

⚠ Rays fail to focus to single point due to <u>non-paraxial effects</u>
↳ <u>Aberrations</u>

<u>Ways to reduce Aberrations:</u> (1) Use higher refractive index (2) Decrease angle of incidence (3) Aspheric surfaces

<u>1^{st} vs 3^{rd} Order theory:</u> Start w/ equating optical path length inside and outside refractive medium



$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right)$$

<u>Task:</u> approximate l_o & l_i to 1^{st} & 3^{rd} order

<u>1^{st} order Theory:</u> $\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$

<u>3^{rd} Order Theory:</u> $\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + h^2\left[\frac{n_2}{2s_o}\left(\frac{1}{s_o} + \frac{1}{R}\right) + \frac{n_2}{2s_i}\left(\frac{1}{R} - \frac{1}{s_i}\right)^2\right]$

⚠ Review Aberration types on slide 31 of Lecture 5

<u>Cauchy's Equation:</u> $\boxed{n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots}$

<u>Axial Chromatic Aberration:</u> $\Delta f(\lambda) = \frac{n(\lambda_0) - n(\lambda)}{n(\lambda) - 1} f_G$ (From lensmaker's Formula)

Note: Chromatic Aberration can be used for phase retrieval!

<u>Abbe Number:</u> $V = \frac{n_D - 1}{n_F - n_C}$

Normal Dispersion: (e.g. in Lenses) refractive index decreases w/ increasing λ

Anomalous Dispersion: (e.g. diffraction gratings) index increases w/ increasing λ

Reduce chromatic aberrations (correct for dispersion) by combining lens + diffraction grating

5 Primary aberrations: (1) Spherical aberration (2) Coma (3) Antistigmatism (4) Field Curvature (5) Distortion

Spherical Aberration: due to Non-paraxiality near edges of lens
↳ Corrected w/ asphere or multiple lenses

Coma: Distortion of off-axis sources → appears to have tail

Astigmatism: Vertical & horizontal have different focus

Field Curvature: Flat objects cannot be fully infocus in flat image Plane

Distortion: Magnification changes as you move away from optical axis.

Resolution, MTF, etc

Resolution: (1) Smallest resolvable object or (2) Smallest distance between objects to be resolvable

Point Spread Function: points in source become "distributions" in image

↳ If imaging system is linear & shift-invariant, then PSF has a convolution relation w/ object:

$$\underset{\text{image}}{I(x,y)} = \underset{\text{object}}{O(x,y)} \otimes \underset{\text{Point Spread Function}}{PSF(x,y)}$$

Three Factors Affect Resolution: (1) wavelength (2) angular aperture of light cone (3) Refractive index surrounding objective/specimen

⚠ Airy disk is the PSF for a clear circular aperture

Rayleigh Resolution Criterion: $R = 1.22\frac{\lambda}{2NA} = \boxed{1.22\frac{\lambda}{2n\sin\theta_{max}}}$

↳ Resolution for a circular aperture

↳ 1.22 corresponds to distance to 1st minimum of Airy disk

Abbe diffraction Limit: highest spatial frequency intensity pattern that can be created by 2 beams interfering

↳ $d = \frac{\lambda}{2NA} = \boxed{\frac{\lambda}{2n\sin\theta_{max}} = d}$

Modulation Transfer Function: (MTF) Characterizes "Resolution" of imaging system, contrasts at each spatial frequency

↳ Fourier transform of PSF

$$PSF = h(x,y) \quad ; \quad MTF = |H(\xi,\eta)|$$

$$H(\xi,\eta) = \iint h(x,y)\, e^{-j2\pi(\xi x+\eta y)}\, dx\,dy$$

$H(\xi,\eta) \equiv$ Optical Transfer Function

Additional E&M Review

Ways to polarize an EM wave: (1) Selective absorbtion (dichroism) (2) Reflection (3) Scattering (4) Birefringence

Rayleigh Scattering: Scattering of light off of Molecules in the air
↳ Scattering is strongly dependent on wavelength → shorter wavelengths scatter more.

Birefringence: Different polarizations have different refractive indices

$\lambda/4$ Wave Plate: Birefringent → converts linear to circular polarization

$\lambda/2$ Wave Plate: Birefringent → converts horizontal linear to vertical linear

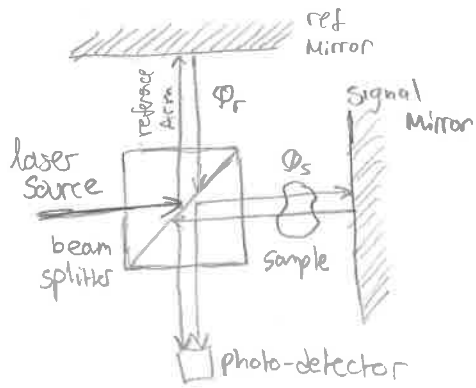
Interference, Diffraction

Intensity of interfering waves: $I = \varepsilon_0 c|E|^2 = \varepsilon_0 c\left(\bar{E}_1+\bar{E}_2\right)+\left(\bar{E}_1+E_2\right)$

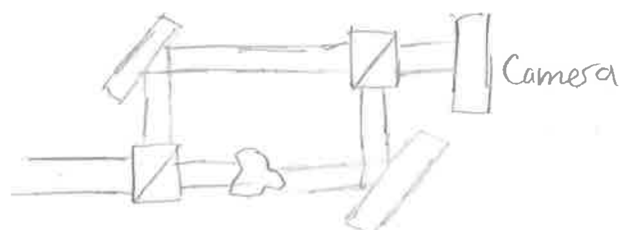
$$= \varepsilon_0 c\left(I_1 + I_2 + 2\bar{E}_1\cdot\bar{E}_2\right)$$

Michelson Interferometer:

$$I_d \propto \left|A_r \exp(i\varphi_r) + A_s \exp(i\varphi_s)\right|^2$$



Mach-Zehnder Interferometer:



Fourier Optics

Huygen's principle: Every point on a wavefront serves as a source of spherical secondary wavelets, such that the primary wavefront at some later time is the envelope of these wavelets.

Fraunhofer diffraction: Far field approximation to spherical wavelets ie $z \to \infty$

Fresnel diffraction: Paraxial approximation to spherical wavelets ie $z \gg x, y$

Huygen's Principle (Math): Propagation is modeled as convolution w/ spherical "phase function" (green's function)

$$g'(x,y) = g(x,y) \otimes h(x,y)$$

Field at x' due to pinhole at x_0: $\frac{\exp(ik(z+l))}{i\lambda l} \exp\left\{ i\pi \frac{(x'-x_0)^2}{\lambda l} \right\}$

Huygens-Fresnel Principle:

$$\boxed{g(x',y') = \frac{z}{i\lambda} \iint g(x,y) \frac{e^{ikr}}{r^2} dx dy}$$

where $r = \sqrt{z^2 + (x'-x)^2 + (y'-y)^2}$

Fraunhofer Approximation:

$$G(x,y) \propto A \iint g(f_x, f_y) \exp\left[-i \frac{2\pi}{\lambda z}(x f_x + y f_y)\right] df_x df_y$$

where $f_x = \frac{x}{\lambda z}$; $f_y = \frac{y}{\lambda z}$

Applies when $z \gg k(f_x^2 + f_y^2)_{max}$ ← This is wrong ⚠

Linearity: $H(g_1) + H(g_2) = H(g_1 + g_2)$; $H(a g_{in}) = a H(g_{in})$

↳ if input is a sum of impulses, then output is a sum of impulses

↳ $g_{out}(x) = \int g_{in}(x') h(x-x') dx'$ ← convolution of input & PSF

⚠ Review End of lecture 17 → Intuitive grasp of far field patterns using knowledge of FFT's

Fresnel Propagation: $G(x',y') = \frac{e^{ikz}}{i\lambda z} \iint dx dy \, g(x,y) \exp\left\{ \frac{ik}{2z}\left[(x'-x)^2 + (y'-y)^2\right] \right\}$

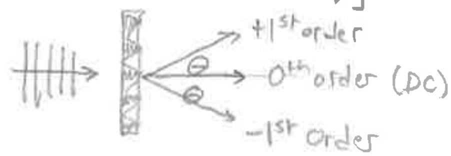
↳ Uses the paraxial Approximation: $R \simeq z\left[1 + \frac{x^2+y^2}{2z^2}\right]$

↳ Back propagate by deconvolving PSF out of field.

Thin transparency approximation: thin transparency is described as a complex transmission function: $g_t(x,y) = a(x,y)\, e^{i\varphi(x,y)}$ ← phase delay; $a(x,y)$ ← attenuation

↳ e.g. Sinusoidal amplitude grating: $g_t(x) = \frac{1}{2}\left[1 + m\cos\left(2\pi \frac{x}{\Lambda} + \varphi\right)\right]$

- ↳ diffraction angle: $\sin\theta \equiv \frac{\lambda}{\Lambda}$
- ↳ Spatial Frequency $u_0 = \frac{1}{\Lambda} = \frac{\sin\theta}{\lambda}$
- ↳ diffraction efficiencies: $\eta_0 = \left(\frac{1}{2}\right)^2$; $\eta_{\pm 1} = \left(\frac{m}{4}\right)^2$



Talbot Effect: Any periodic function repeats itself along z w/ propagation ⟹ Self imaging

⚠ Show that at certain distances behind an object, the periodic transmittance function will be reproduced. (Hint: Show that PSF = const wrt x & y at certain z's)

Talbot distance: $Z_n = \frac{2nL^2}{\lambda}$ ← Self imaging, $n = 1, 2, \dots$ — $L \equiv$ Period of object/grating

2f FT lens System: $g_{out}(x', y'; f) \propto \iint g_{in}(x,y) \exp\left\{-i2\pi\left[x\frac{x'}{\lambda f} + y\frac{y'}{\lambda f}\right]\right\} dx\,dy$

Auto correlation: $g(x) * g(x) = \int g(x')\, g(x'-x)$

↳ $\mathcal{F}\{g * g\} = \tilde{G}\tilde{G}^* = |G|^2$ ← Power spectrum

Parseval's Theorem: $\int_{-\infty}^{\infty} |g(x)|^2 dx = \int_{-\infty}^{\infty} |\tilde{G}(f_x)|^2 df_x$

4F System: