# Equations To Remember:

Threshold Gain: $g_{th} = \frac{\alpha_i + \alpha_m}{\Gamma} = \boxed{\frac{\alpha_i}{\Gamma} + \frac{1}{2\Gamma L} \ln \frac{1}{R_1 R_2}}$

Quantum Efficiency: $QE = \frac{\alpha_m}{\alpha_i + \alpha_m}$ Quality factor: $\boxed{Q = \frac{\omega_0}{\Delta\omega} = \omega_0 \tau_p}$ $\frac{n}{c\alpha_{tot}}$

Energy + Momentum Conservation: $E_g + \frac{\hbar^2 k^2}{2m_r^*} = h\nu$

Electron Concentration: $n = \int_{E_c}^{\infty} dE\, f_n(E) \rho(E)$ where $f_n(E) = \frac{1}{1 + e^{(E-F_n)/kT}}$

Electron Concentration in 3D: $\boxed{\begin{cases} F_n \ll E_c \Rightarrow n = N_c\, e^{F_n - E_c/kT} \\ F_n \gg E_c \Rightarrow n = N_c \frac{4}{3\sqrt{\pi}} \left(\frac{F_n - E_c}{kT}\right)^{3/2} \end{cases}}$

↳ Effective DOS: $N_c = 2\left(\frac{\pi m_e kT}{2\pi^2 \hbar^2}\right)^{3/2}$

Electron Concentration in 2D: $\boxed{\begin{cases} F_n \ll E_{e1} \Rightarrow N \approx \left(\frac{m_e kT}{\pi\hbar^2 L_z}\right) e^{-\frac{E_{e1} - F_c}{kT}} \\ F_n \gg E_{e1} \Rightarrow N = \frac{m_e^*}{\pi\hbar^2 L_z}(F_n - E_{e1}) \end{cases}}$

↳ Exact: $N^{2D} = \left(\frac{m_e kT}{\pi\hbar^2 L_z}\right) \ln\left[1 + e^{-\frac{E_{e1} - F_c}{kT}}\right]$

Density of States in 3D: $\boxed{\rho_e^{3D}(E) = \frac{m_e^*}{\hbar^2 \pi^2} \sqrt{\frac{2m_e^*(E - E_c)}{\hbar^2}}}$

in 2D: $\boxed{\rho_e^{2D}(E) = \frac{m_e^*}{\pi\hbar^2 L_z} \sum_{m=1}^{\infty} H(E - E_m)}$ $E_m = E_c + \frac{\hbar^2 \pi^2}{2m_e^*}\left(\frac{m}{L_z}\right)^2$

in 1D: $\rho_e^{1D}(E) = \frac{1}{4\pi^3 L_x L_y} \sqrt{\frac{m_e}{2\hbar}} \sum_{m,n} \frac{1}{\sqrt{E - E_c - E_{mn}}}$

in 0D: $\rho_e^{0D}(E) = \frac{2}{V} \sum_{m,n,q} \delta_{E, E_c + E_{mnq}}$

Schrodinger's Equation: $\hat{H}\Psi(\bar{r},t) = i\hbar \frac{\partial}{\partial t}\Psi(\bar{r},t)$ where $\hat{H} = \frac{p^2}{2m} + V(\bar{r},t)$ ; $\bar{p} = i\hbar\bar{\nabla}$

Infinite Square Well: $\Psi(\bar{r},t) = e^{i(k_x x + k_y y)} e^{-i\omega t} \begin{cases} A\sin(k_z z) \\ A\cos(k_z z) \end{cases}$, $\boxed{E_n = \frac{\hbar^2}{2m_e^*}\left(\frac{n\pi}{L_z}\right)^2}$

Gaussian Beam: $\boxed{w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}}$ ; $\boxed{z_0 = \frac{\pi w_0^2}{\lambda}}$ ; $\boxed{R(z) = z\left(1 + \left(\frac{z_0}{z}\right)^2\right)}$

Fermi's Golden Rule: $\boxed{W_{i\to f} = \frac{2\pi}{\hbar}|\mathcal{H}'_{fi}|^2\delta(E_f - E_i - \hbar\omega) + \frac{2\pi}{\hbar}|\mathcal{H}'^{+}_{fi}|^2\delta(E_f - E_i + \hbar\omega)}$

↳ FGR for Many Energy levels: $W_{i\to f} = \frac{2\pi}{\hbar}|\mathcal{H}'_{fi}|^2 \underline{\rho_f(E_f)}\,\delta(E_f - E_i - \hbar\omega) + \cdots$

Matrix Element: $\boxed{\mathcal{H}'_{ba} = -\frac{eA_0}{2m_0}\hat{e}\cdot\langle b|\bar{P}|a\rangle}$

Dipole Approximation: $\boxed{\mathcal{H}'_{ba} \simeq -e\bar{E}\cdot\langle b|\bar{r}|a\rangle}$

Absorption/Gain Spectrum: $\boxed{\alpha^{3D}(\hbar\omega) = -\frac{2\pi}{\hbar}|\mathcal{H}'_{ba}|^2\rho_r^{3D}(\hbar\omega - E_g)\,f_g(\hbar\omega - E_g)}$

$$\alpha^{2D}(\hbar\omega) = -\frac{2\pi}{\hbar}|\mathcal{H}'_{ba}|^2\rho_r^{2D} f_g(\hbar\omega - E_g)$$

Bernard-Durafourg gain condition: $\boxed{E_g < \hbar\omega < \Delta F}$

Excess Electron/Hole Energy: $\begin{cases} E_b - E_c = (\hbar\omega - E_g)\dfrac{m_r^*}{m_e^*} \\ E_v - E_a = (\hbar\omega - E_g)\dfrac{m_r^*}{m_h^*} \end{cases}$

Spontaneous Emission Rate: $\boxed{r_{21}^{spon}(E_{21}) = \frac{8\pi n^2 E_{21}^2}{h^3 c^3} C_0 |\hat{e}\cdot\bar{P}_{cv}|\,\rho_r(\hbar\omega - E_g)\frac{f_e}{f_g}}$

Bose-Einstein Distribution: $\dfrac{1}{e^{\hbar\omega/kT} - 1}$

Lorentzian: $\boxed{g(\Delta E) = \frac{1}{\pi}\frac{\frac{\Gamma}{2}}{\Delta E^2 + (\frac{\Gamma}{2})^2}}$

Maximum Intersubband Transition: $\alpha_{max} = \frac{\pi\omega}{nc\varepsilon_0}\frac{1}{\pi}\frac{1}{\Gamma/2}\frac{m_e^*}{\pi\hbar^2 L_z}E_{21}^c|\mu_{z_1}|^2$

Diode IV Characteristics: $\boxed{I = I_0(e^{qV/kT} - 1)}$ $\quad I_0 = q\left(\frac{D_n}{L_n}n_{po} + \frac{D_p}{L_p}P_{no}\right)$

V Number: $\boxed{V = k_0 d\sqrt{n_1^2 - n_2^2}}$ Single Mode Condition: $\boxed{V < \pi}$

Confinement Factor: $\boxed{\Gamma \approx \frac{V^2}{2 + V^2}}$

Effective Index: $k_z = n_{eff} k_0$

Cavity Modes: $\overset{\text{integer}}{m}\dfrac{c}{2n_{eff}L} = f$ Mode Spacing: $\boxed{\Delta f = \dfrac{c}{2n_{eff}L}\dfrac{1}{1+\dfrac{f}{n_{eff}}\dfrac{dn_{eff}}{df}}}$

Rate Equations:
$$\boxed{\begin{cases} \dfrac{dN}{dt} = \dfrac{\eta_i I}{qV} - \dfrac{N}{\tau} - g(N)v_g S \\ \dfrac{dS}{dt} = \Gamma g(N) v_g S - \dfrac{S}{\tau_p} + \Gamma\beta R_{sp} \end{cases}}$$

$\dfrac{dN}{dt}$ = Amount of charge injected − recombining charge − charge lost due to stimulated emission

$\dfrac{dS}{dt}$ = Photons gained due to stimulated emission − Photons lost due to lifetime + Photons gained due to spontaneous emission

Threshold Current: $I_{th} = \dfrac{qV}{\eta_i}\dfrac{N_{th}}{\tau}$

Output Power above Threshold: $\boxed{P_{out} = \dfrac{\hbar\omega}{q}\eta_i\eta_o(I - I_{th})}$

Relaxation Oscillation Frequency: $\boxed{\omega_R^2 = \dfrac{v_g a S}{\tau_p}}$

Direct Modulation Transfer function: $\boxed{H(\omega) = \dfrac{\omega_R^2}{\omega_R^2 - \omega^2 + j\omega\gamma}}$

Transparency Condition: $\Delta F = E_g$

Ideal QW Transparency concentration: $N_{tr}^{ideal} = \dfrac{m_e^*}{\pi\hbar^2 L_z} k_B T \ln 2$

Strain Trend: lattice spacing ↑ $E_g$ ↓, lattice spacing ↓ $E_g$ ↑

⚠ Want Compressive strain for Lasers, Tensile strain for Amplifiers

P-i-n Photocurrent: $\boxed{I_{ph} = \eta\dfrac{q}{\hbar\omega}P_{opt}}$

Ramo's Theorem: Current only exists while charge is moving) $i(t) = \dfrac{qv(t)}{d}$

P-i-n Temporal Resonse: $\boxed{\tau = \dfrac{d}{v_h} + R\dfrac{\epsilon A}{d}}$

P-i-n Efficiency: $\boxed{\eta = \eta_i(1 - e^{-\alpha d})}$

p-i-n 3dB frequency: $\boxed{f_{3dB} = \frac{v_h}{2\pi}\frac{1}{d}}$

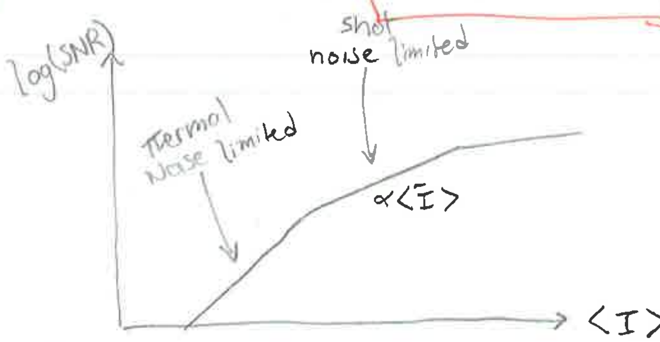
p-i-n Bandwidth-Efficieicy tradeoff: $\boxed{\eta f_{3dB} \simeq \frac{\alpha \eta_i v_h}{2\pi}}$

Photoconductor Current: $\Delta I = P_{opt}\left[\eta \frac{q}{\hbar\omega}\right]\frac{\tau_n}{\tau_t}$

Photoconductive Gain: $\boxed{\frac{\tau_n}{\tau_t}}$

Photoconductor 3dB Bandwidth: $\boxed{f_{3dB} = \frac{1}{2\pi}\frac{1}{\tau_n}}$

Shot Noise: $\boxed{\langle i_n^2 \rangle = 2q\langle I \rangle d\nu}$ Thermal Noise: $\boxed{\langle i_{th}^2 \rangle = \frac{4kT\,d\nu}{R}}$

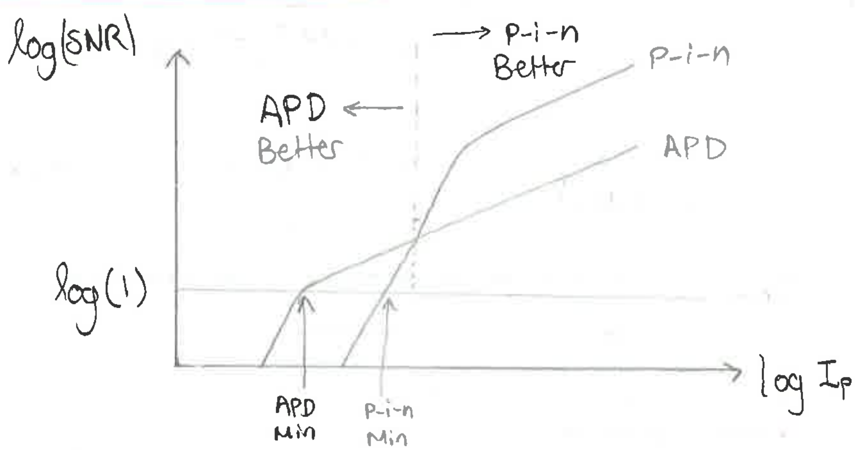

Multiplication Factor: $\boxed{M = \frac{J}{J_n(0)}} = \frac{1-k}{e^{-(1-k)\alpha_n W} - k}$ $\boxed{k = \frac{\beta_e}{\alpha_n} < 1}$

$k = 1 \Rightarrow$ instability

Noise Factor: $\boxed{F = \frac{\langle M^2 \rangle}{\langle M \rangle^2}} = k\langle M_n \rangle + (1-k)\left(2 - \frac{1}{\langle M_n \rangle}\right)$

$$\boxed{SNR = \frac{\frac{1}{2} I_{ph}^2 \langle M \rangle^2}{2q(I_{ph} + I_{dark})F\langle M \rangle^2 \Delta\nu + 4kT\Delta\nu/R}}$$

Note: $\boxed{\langle M \rangle^2 = 1 = F}$ for Photodiode



Snells law : $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$

Femat's Principle : $\boxed{OPL = \int_{path} n(r)\,ds}$ is minimized

Critical angle : $n_1 \sin\theta_{crit} = n_2$

Spherical ("curved") surface : $\boxed{\begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R n_2} & \frac{n_1}{n_2} \end{pmatrix}} = A(R)$

Thin lens : $\boxed{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}} = A(R_2)\,A(R_1)$

Lensmaker's Formula : $\boxed{\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

Image Formation : lateral Magnification : $M_T \equiv \frac{h_i}{h_o} = -\frac{f}{x_o} = -\frac{x_i}{f}$

Imaging Condition : $x_o x_i = f^2 \Rightarrow \frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$

Numerical Aperture : $\boxed{NA = n \sin\theta}$ f-number : $\boxed{f/\# = \frac{1}{2NA}}$

Cauchy's Equation : $n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$

Abbe Number : $V = \frac{n_D - 1}{n_F - n_C}$

Point Spread Function : $\boxed{I(x,y) = O(x,y) \otimes PSF(x,y)}$ Image

Rayleigh Resolution Criterion : $\boxed{R = 1.22 \frac{\lambda}{2n \sin\theta_{max}}}$

Abbe diffraction Limit : $\boxed{d = \frac{\lambda}{2n\sin\theta_{max}}}$

Optical Transfer Function : $\mathcal{F}\{PSF(x,y)\} = H(f_x, f_y)$

Modulation transfer function : $MTF = |H(f_x, f_y)|$

Huygens-Fresnel Principle: $\boxed{g(x',y') = \frac{z}{i\lambda}\iint dx\,dy\, g(x,y)\,\frac{e^{ikr}}{r^2} \quad r = \sqrt{z^2 + (x'-x)^2 + (y'-y)^2}}$

Fraunhofer Approximation: $\boxed{G(x',y') = \frac{e^{ikz}\, e^{i\frac{k}{2z}(x'^2+y'^2)}}{i\lambda z}\iint dx\,dy\, g(x,y)\, e^{-i\frac{2\pi}{\lambda z}(x'x+y'y)} \quad z \gg \frac{k(x^2+y^2)}{2}}$

Linearity: $H(g_1) + H(g_2) = H(g_1+g_2)$ ; $H(a g_{in}) = a H(g_{in})$

Fresnel Propagation: $\boxed{G(x',y') = \frac{e^{ikz}}{i\lambda z}\iint dx\,dy\, g(x,y)\, e^{i\frac{k}{2z}[(x'-x)^2+(y'-y)^2]}}$

Talbot Distance: $\boxed{Z_n = \frac{2nL^2}{\lambda}}$ $n = 1, 2, \ldots$ $L \equiv$ period of object grating

2f FT lens System: $\boxed{g_{out}(x',y';f) \propto \iint g_{in}(x,y)\, e^{-i2\pi\left(x\frac{x'}{\lambda f} + y\frac{y'}{\lambda f}\right)} dx\,dy}$

4F System

$g_{out} = g_{in}\left(-\frac{f_2}{f_1}x\right)$

Autocorrelation: $g(x) \overset{\text{correlation}}{\star} g(x) = \int g(x')\, g(x'-x)\, dx'$

Power Spectrum: $\mathcal{F}\{g \star g\} = \tilde{G}\tilde{G}^* = |G|^2$

Parseval's Theorem: $\boxed{\int_{-\infty}^{\infty} |g(x)|^2 dx = \int_{-\infty}^{\infty} |\tilde{G}(f_x)|^2 df_x}$

Fresnel Propagation as Fourier Transforms: $G(x',y') = g(x,y) \otimes h(x,y;z)$

where $h(x,y;z) = \frac{e^{ikz}}{i\lambda z}\, e^{i\frac{k}{2z}[(x'-x)^2+(y'-y)^2]}$

Thus

$$G(x',y') = \mathcal{F}^{-1}\left\{ \mathcal{F}\{g(x,y)\}\, H(u,v) \right\}$$

where

$$H(u,v) = \mathcal{F}\{h(x,y;z)\} = \frac{e^{ikz}}{i\lambda z}\, e^{-i\pi\lambda z(u^2+v^2)}$$