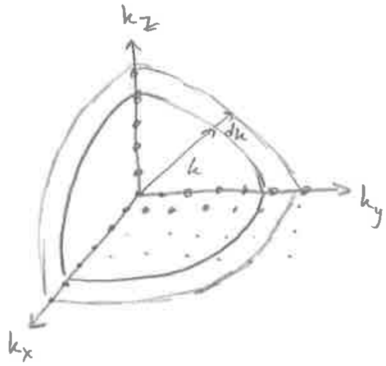
Density of States in 3D



Number of states in a shell of radius k and infinitesimal thickness dk:

⚠ Pauli Exclusion

$$dN = 4\pi k^2 dk \times \frac{1}{\underbrace{\frac{(2\pi)^3}{V}}_{\text{Volume of single state in k-space}}} \times 2$$

We wish to express this "density of states" as a function of energy. Let us consider the DOS for an electron in the conduction band:

$$E_n = E_c + \frac{\hbar^2 k^2}{2m_e}$$

It follows that:

$$k = \sqrt{\frac{2m_e(E-E_c)}{\hbar^2}} \Rightarrow dk = \sqrt{\frac{m_e}{2\hbar^2}} \frac{1}{\sqrt{E-E_c}} dE$$

Thus:

$$dN = \frac{V}{\pi^2} \frac{2m_e(E-E_c)}{\hbar^2} \sqrt{\frac{m_e}{2\hbar^2}} \frac{dE}{\sqrt{E-E_c}}$$

$$= \frac{4V}{\pi^2} \left(\frac{m_e}{2\hbar^2}\right)^{3/2} \sqrt{E-E_c}\, dE$$

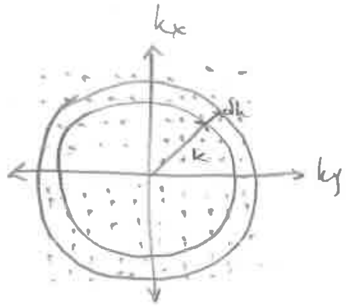
Hence the density of states is:

$$\frac{dN}{dE} = \frac{\sqrt{2}V}{\pi^2} \left(\frac{m_e}{\hbar^2}\right)^{3/2} \sqrt{E-E_c}$$

Generally, we want to normalize this quantity by the Volume:

$$\boxed{\frac{dN}{dE} = \frac{m_e}{\pi \hbar^2} \sqrt{\frac{2m_e(E-E_c)}{\hbar^2}}}$$

Density of States in 2D



The area of a thin shell in k space is:

$$dA_k = 2\pi k dk \times 2$$

The Area of a single state is $\frac{(2\pi)^2}{L_x L_y}$ Thus the number of states in that thin shell is:

$$dN = \frac{4\pi k\, dk\, L_x L_y}{(2\pi)^2} = \frac{L_x L_y}{\pi} k\, dk$$

Normalizing by volume:

$$dN = \frac{1}{L_z \pi} k dk$$

Recall that for a conduction electron in a 2D Quantum confined structure:

$$E = E_c + E_m + \frac{\hbar^2 k^2}{2m_e}$$

$$\Rightarrow \quad k = \sqrt{\frac{2m_e(E-E_c-E_m)}{\hbar^2}} \quad \Rightarrow \quad dk = \sqrt{\frac{m_e}{2\hbar^2}} \frac{dE}{\sqrt{2m_e(E-E_c-E_m)}}$$

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Thus,

$$dN = \frac{1}{\pi L_z} \sqrt{\frac{2m_e}{\hbar^2}} \sqrt{\frac{m_e}{2\hbar^2}}\, dE$$

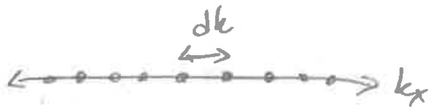
Hence,

$$\frac{dN_{2D}}{dE} = \frac{m_e}{\pi \hbar^2 L_z}$$

This solution is limited to the case in which only the 1st energy level is occupied. As the Energy increases, additional energy levels of the quantum well become accessible. This increases the DOS:

$$\boxed{\frac{dN_{2D}}{dE} = \frac{m_e}{\pi \hbar^2 L_z} \sum_{m=1}^{\infty} H(E-E_m)} \quad \text{where } E_m = \frac{\hbar^2 \pi^2}{2m_e^*}\left(\frac{m}{L_z}\right)^2$$

Density of States in 1D



In 1D the differential "Volume" is:

$$dL = dk$$

Thus the number of states contained w/in the differential "Volume" is:

$$dN = \frac{dk}{\frac{(2\pi)^3}{L_x}} = 2L_x \frac{dk}{(2\pi)^3}$$

Normalized by Volume:

$$dN = \frac{dk}{4\pi^3 L_y L_z}$$

The E-k relationship for a quantum wire is:

$$E = E_c + E_{m,n} + \frac{\hbar^2 k_x^2}{2m_e} \quad \text{where } E_{mn} = \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{m}{L_y}\right)^2 + \left(\frac{n}{L_z}\right)^2\right]$$

$$\Rightarrow \quad k_x = \sqrt{\frac{2m_e(E-E_c-E_{mn})}{\hbar^2}} \quad \Rightarrow \quad dk_x = \sqrt{\frac{m_e}{2\hbar^2}} \frac{dE}{\sqrt{E-E_c-E_{mn}}}$$

Hence the DOS becomes:

$$\frac{dN}{dE} = \frac{1}{4\pi^3 L_x L_y}\sqrt{\frac{m_e}{2\hbar^2}} \frac{1}{\sqrt{E-E_c-E_{mn}}}$$

Notice that, like the 2D case, as the energy increases, additional Energy levels become accessible which increases the available DOS:

$$\boxed{\frac{dN_{1D}}{dE} = \frac{1}{4\pi^3 L_x L_y}\sqrt{\frac{m_e}{2\hbar^2}} \sum_{m,n} \frac{1}{\sqrt{E-E_c-E_{mn}}} H(E-E_c-E_{mn})}$$

Density of states in 0D

In a quantum dot, electrons are completely confined in each spatial direction. This means that for each energy level, the density of states is simply:

$$\frac{dN}{dE} = 2 \Rightarrow \frac{dN}{dE} = \frac{2}{V} \quad \text{(Normalized by volume of QD)}$$

Electrons in a QD are constrained to very specific energies. The DOS thus increments accordingly:

$$\boxed{\frac{dN_{0D}}{dE} = \frac{2}{V}\sum_{m,n,q} \delta_{E, E_c + E_{mnq}}}$$

where

$$E_{mnq} = \frac{\hbar^2\pi^2}{2m_e}\left[\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2 + \left(\frac{q}{L_z}\right)^2\right]$$