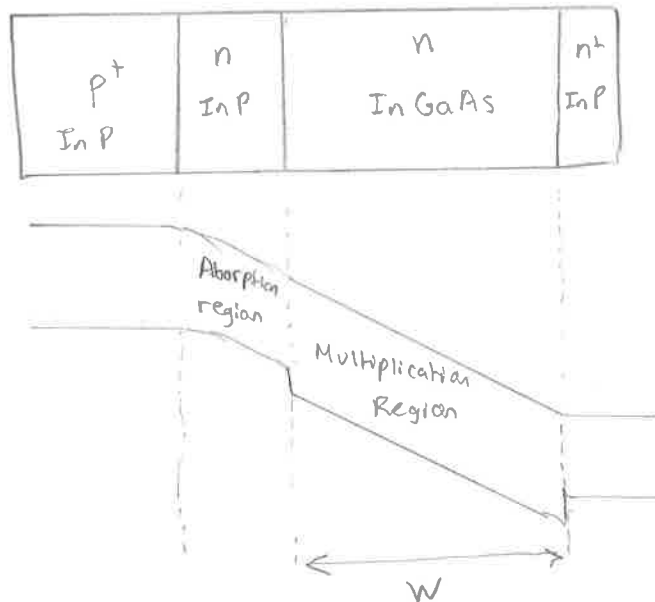


Avalanche Photodiodes

Consider the structure of a Separate Absorption & Multiplication APD:



A smaller bandgap heavily doped region captures photons, generating electron hole pairs which are swept into the multiplication region by the built in electric field. The multiplication region, which has a wider bandgap such that it is not absorbing, is long, allowing carriers to accelerate to high enough energies such that avalanching can occur.

The amount of multiplication is quantified by an "ionization" coefficient, since every ionized electron is accompanied by an ionized hole, the ionization of one carrier will also result in the ionization of the other carrier.

From this logic, it follows that the current density must obey:

$$\frac{dJ_n(x)}{dx} = \alpha_n J_n(x) + \beta_p J_p(x) \quad (1)$$

Since holes are accelerated in the opposite direction:

$$\frac{dJ_p(x)}{dx} = \frac{d(-J_n(x))}{dx} \Rightarrow -\frac{dJ_p(x)}{dx} = \alpha_n J_n(x) + \beta_p J_p(x) \quad (2)$$

From this, it is evident That:

$$\frac{d}{dx} (J_p(x) + J_n(x)) = 0 \Rightarrow J_p(x) + J_n(x) = \text{const} = J$$

We can thus decouple the two differential equations:

$$J_p(x) = J - J_n(x)$$

So,

$$\frac{dJ_n(x)}{dx} = \alpha_n J_n(x) + \beta_p [J - J_n(x)] = (\alpha_n - \beta_p) J_n(x) + \beta_p J$$

$$\Rightarrow \frac{dJ_n(x)}{dx} - (\alpha_n - \beta_p) J_n(x) = \beta_p J$$

$$\Rightarrow \gamma \frac{dJ_n(x)}{dx} - \gamma (\alpha_n - \beta_p) J_n(x) = \gamma \beta_p J$$

Try:

$$\frac{d}{dx} (\gamma J_n) = \gamma \frac{dJ_n(x)}{dx} + \frac{d\gamma}{dx} J_n(x)$$

Which implies:

$$\frac{d\gamma}{dx} = -(\alpha_n - \beta_p) \gamma \Rightarrow \gamma = e^{-(\alpha_n - \beta_p)x}$$

So,

$$\frac{d}{dx} (e^{-(\alpha_n - \beta_p)x} J_n(x)) = \beta_p J e^{-(\alpha_n - \beta_p)x}$$

$$\Rightarrow e^{(\alpha_n - \beta_p)x} J_n(x) = \frac{\beta_p J}{\beta_p - \alpha_n} \left[e^{-(\alpha_n - \beta_p)w} - 1 \right]$$

$$\Rightarrow J_n(x) = \frac{\beta_p J}{\beta_p - \alpha_n} \left[e^{-(\alpha_n - \beta_p)w} - 1 \right] e^{+(\alpha_n - \beta_p)x} + C e^{(\alpha_n + \beta_p)x}$$

Integrate
 x from
0 to w

Constant of
integration

We use the boundary condition $J_n(w) = J$ since we assume that the total current leaving the junction at w is entirely electrons.

$$J = \frac{\beta_p}{\beta_p - \alpha_n} \left[e^{-(\alpha_n - \beta_p)W} - 1 \right] e^{(\alpha_n - \beta_p)W} + c e^{(\alpha_n - \beta_p)W}$$

$$C = J e^{-(\alpha_n - \beta_p)W} - J \frac{\beta_p}{\beta_p - \alpha_n} \left[e^{-(\alpha_n - \beta_p)W} - 1 \right]$$

Eventually, it can be shown that the Multiplication factor (gain) is given by:

$$M_n = \frac{1-k}{e^{-(1-k)\alpha_n W} - k} \quad \text{where } k = \frac{\beta_p}{\alpha_n}$$

The response time of an APD is given by:

$$\tau = \tau_t + \tau_m$$

where τ_t is the transit time:

$$\tau_t = \frac{W_{abs}}{v_n}$$

and τ_m is the multiplication time:

$$\tau_m = \frac{M_n k W}{v_e} + \frac{W}{v_n} \approx \frac{M_n k W}{v_e}$$

The gain bandwidth product is given by:

$$G \times BW = M_n \times \frac{1}{2\pi\tau_m} = \frac{v_e}{2\pi k W}$$

Next, noise causes fluctuations in the gain. As a result, noise gets amplified. This effect is quantified by the noise Factor F :

$$F = \frac{\langle M^2 \rangle}{\langle M \rangle^2} = k \langle M_n \rangle + (1-k) \left(2 - \frac{1}{\langle M_n \rangle} \right)$$

The noise Figure is more commonly used and is given by:

$$NF = 10 \log F$$

Finally, because of the gain, APD's SNR is rather different than the SNR of a P-i-N. In particular, both the signal & shot noise are amplified:

$$SNR = \frac{\frac{1}{2} \left(\eta_i P_{opt} \frac{q}{h\nu} \right)^2 \langle M \rangle^2}{2q(I_p + I_s + I_0) R \langle M \rangle^2 \Delta\nu + \frac{4kT\Delta\nu}{R}}$$